

Acetaminophen

The objective of this activity is to use algebraic representations to build recursive equations that represent the levels of acetaminophen in the body.

Headaches are common concerns for many individuals. There are numerous over-the-counter medications that are available to relieve headaches, including **acetaminophen**.

It is theorized that acetaminophen blocks an enzyme that sends out chemicals that make our bodies feel pain. Once entering your body, like many other medications, acetaminophen is processed in the liver into harmless substances and removed from your body.



Half-Life

The **half-life** of acetaminophen in healthy young adults is approximately 2.5 hours. This means that it takes about 2.5 hours for the concentrations of acetaminophen to decrease by 50%.

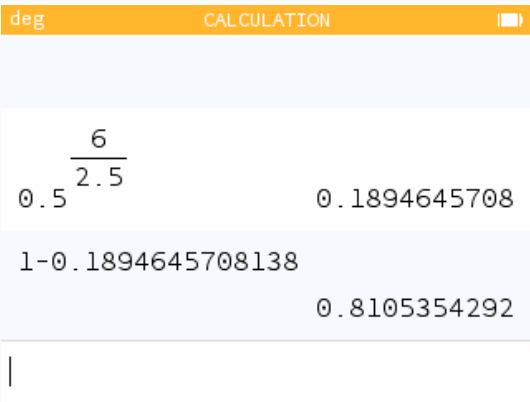
1. What percent of acetaminophen is removed from the body after 4 hours?

To determine the percent of a substance with a half-life of T hours remaining after t hours, we use the formula $P = \frac{1}{2}^{t/T}$.

deg	CALCULATION	
	$0.5^{\frac{4}{2.5}}$	0.3298769777
	$1 - 0.32987697769322$	0.6701230223

67% of acetaminophen is removed from the body after 4 hours.

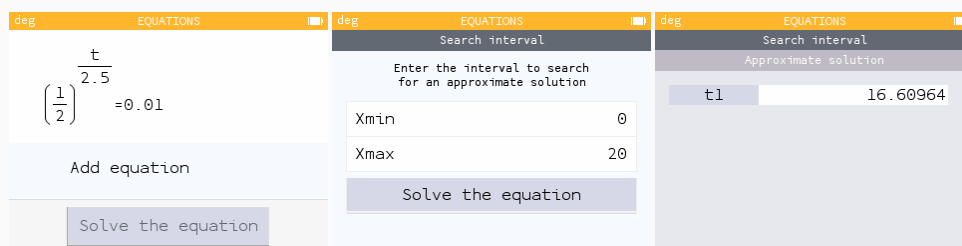
2. What percent of acetaminophen is removed from the body after 6 hours?



The screenshot shows the NUMWORKS CALCULATOR app in 'deg' mode. The display shows the calculation $0.5^{\frac{6}{2.5}}$ resulting in 0.1894645708 . Below this, the calculation $1 - 0.1894645708138$ is shown, resulting in 0.8105354292 . The text below the calculator states: "81% of acetaminophen is removed from the body after 6 hours."

3. How long will it take for 99% of the acetaminophen to be removed from the body?

If 99% of the acetaminophen is removed, only 1% remains. To find the time t , use the Equations app to solve $\frac{1}{2}^{t/2.5} = 0.01$.



The three screenshots show the NUMWORKS EQUATIONS app. The first screenshot shows the equation $\left(\frac{1}{2}\right)^{\frac{t}{2.5}} = 0.01$ entered. The second screenshot shows the 'Search interval' screen with 'Xmin' set to 0 and 'Xmax' set to 20. The third screenshot shows the 'Approximate solution' screen with the solution $t1 = 16.60964$.

It will take 16.6 hours for 99% of acetaminophen to be removed from the body.

Regular and Extra Strength

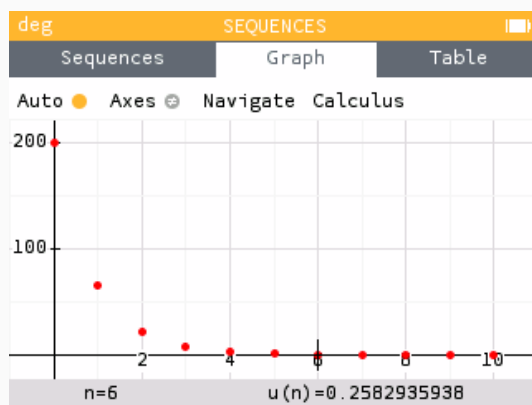
For a common brand of acetaminophen, one "Regular Strength" capsule contains 100mg of acetaminophen and the "Extra Strength" capsules each contain 500mg of acetaminophen.

- To relieve her headache, Ranya decides to take two "Regular Strength" capsules. Write a set of recursive equations that represent the amount of acetaminophen in Ranya's body at the end of each four-hour period.

The screenshot shows the NUMWORKS SEQUENCES app interface. At the top, there are tabs for 'Sequences', 'Graph', and 'Table'. The 'Sequences' tab is selected. Below the tabs, the recursive formula $u_{n+1} = u_n - 0.67u_n$ is entered in red. Below that, the initial value $u_0 = 200$ is entered in red. A blue button labeled 'Add sequence' is visible. At the bottom, there are two buttons: 'Plot graph' and 'Display values'. Below the app interface, the text 'where n is measured in 4-hour periods' is displayed.

2. How much acetaminophen will remain in her system 24 hours after taking the 200mg dose?

To determine the amount of acetaminophen in her body after 24 hours, we look at u_6 .



Since $u_6 = 0.258$, she will only have 0.258 mg of acetaminophen left in her body.

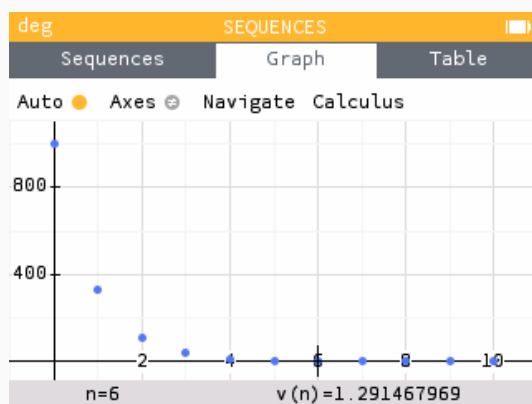
3. Ehab prefers to take two "Extra Strength" capsules. Write a set of recursive equations that represent the amount of acetaminophen in Ehab's body at the end of each four-hour period.

The screenshot shows the NUMWORKS SEQUENCES app interface. At the top, there are tabs for 'Sequences', 'Graph', and 'Table'. The 'Sequences' tab is selected. Below the tabs, the recursive formula $v_{n+1} = v_n - 0.67v_n$ is entered in blue. Below that, the initial value $v_0 = 1000$ is entered in blue. A blue button labeled 'Add sequence' is visible. At the bottom, there are two buttons: 'Plot graph' and 'Display values'.

where n is measured in 4-hour periods

4. How much acetaminophen will remain in his system 24 hours after taking the 1000mg dose?

To determine the amount of acetaminophen in his body after 24 hours, we look at v_6 .



Since $v_6 = 1.291$, he will only have 1.291 mg of acetaminophen left in his body.

Therapeutic Level

The **therapeutic level** of a drug in the bloodstream is the range within which that drug is expected to be effective. Acetaminophen has a therapeutic level of $10\text{-}20\mu\text{g/mL}$.

1. The adult human body has approximately 5L of blood. Determine the amount of acetaminophen needed in the body to feel the effects.

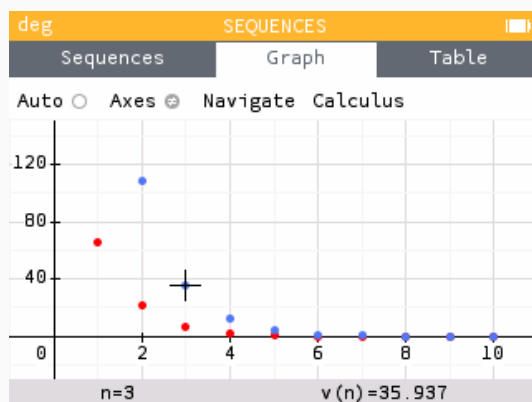
To determine the amount of acetaminophen needed, we multiply the amount of blood in the body by the lower and upper values of the therapeutic level range.

deg CALCULATION	
$5\text{L} \frac{10\text{ }\mu\text{g}}{\text{mL}} \rightarrow \text{mg}$	50_mg
$5\text{L} \frac{20\text{ }\mu\text{g}}{\text{mL}} \rightarrow \text{mg}$	100_mg

The acetaminophen is effective when 50-100mg are in the body.

2. Approximately how long will Ranya's "Regular Strength" medication be effective? How much longer will Ehab's "Extra Strength" medication be effective?

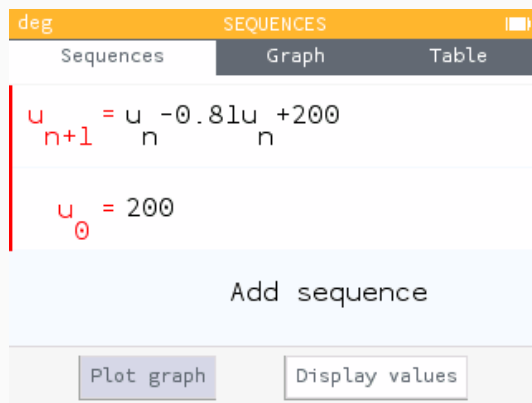
Once the amount of acetaminophen drops below 50mg, the medication is no longer effective. For Ranya's "Regular Strength" medication, this occurs between $n=2$ and $n=3$. Therefore the medication is no longer effective sometime between 8 and 12 hours. Ehab's "Extra Strength" medication drops below 50mg between $n=3$ and $n=4$. Therefore his medication is effective for approximately 4 hours longer than Ranya's.



Continued Use

Both medications recommend taking two capsules **every 6 hours**.

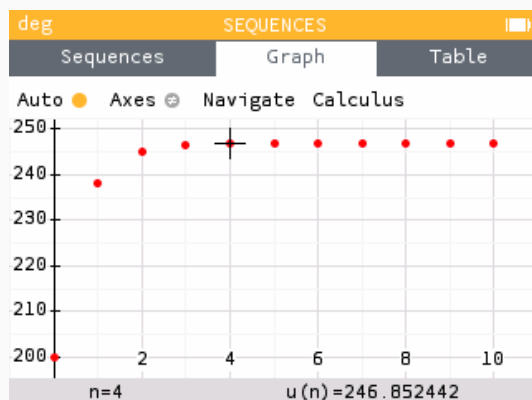
1. If Ranya takes an additional 200mg every six hours after her initial dose, write a set of recursive equations that represent the amount of acetaminophen in her body at the end of each six-hour period.



where n is measured in 6-hour periods

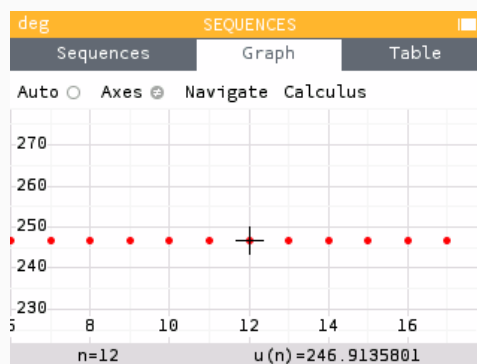
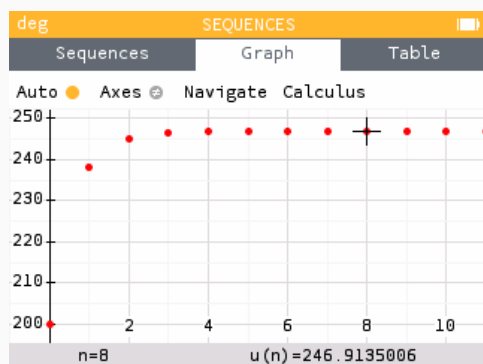
2. How much acetaminophen will remain in her system 24 hours after taking the initial 200mg dose? 48 hours? 72 hours?

To determine the amount of acetaminophen in her body after 24 hours, we look at u_4 .



Since $u_4 = 246.85$, she will only have 246.85 mg of acetaminophen left in her body.

Similarly, to determine the amount of acetaminophen in her body after 48 and 72 hours, we look at u_8 and u_{12}



We see that we have reached an equilibrium value of approximately 246.91 mg.

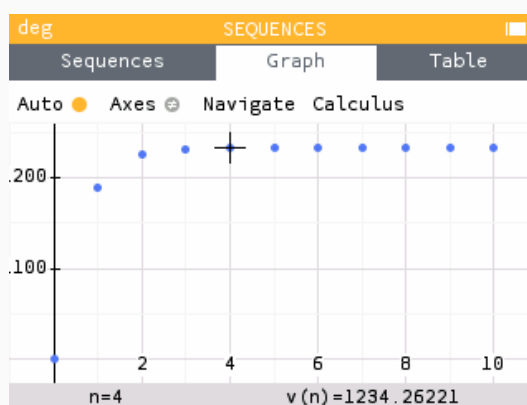
3. If Ehab takes an additional 1000mg every six hours after his initial dose, write a set of recursive equations that represent the amount of acetaminophen in his body at the end of each six-hour period.

The screenshot shows the NUMWORKS SEQUENCES interface. The 'Sequences' tab is selected. The recursive formula entered is $v_{n+1} = v_n - 0.81v_n + 1000$. The initial condition is $v_0 = 1000$. Below the formula, there is a button labeled 'Add sequence'. At the bottom, there are two buttons: 'Plot graph' and 'Display values'.

where n is measured in 6-hour periods

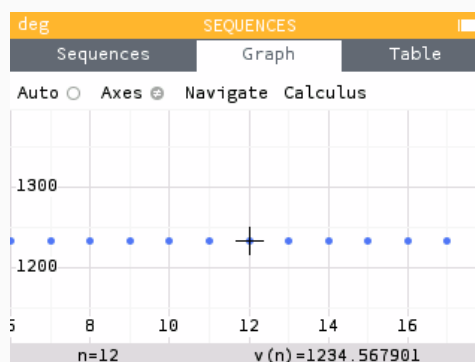
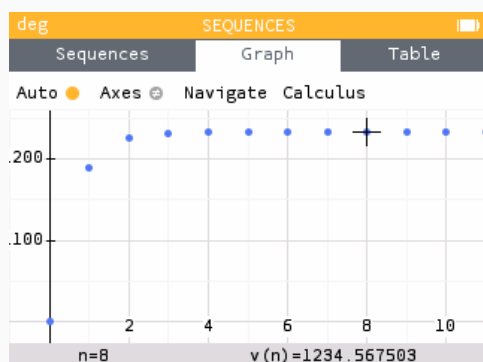
4. How much acetaminophen will remain in his system 24 hours after taking the initial 1000mg dose? 48 hours? 72 hours?

To determine the amount of acetaminophen in his body after 24 hours, we look at v_4 .



Since $v_4 = 1234.26$, he will only have 1234.26 mg of acetaminophen left in his body.

Similarly, to determine the amount of acetaminophen in his body after 48 and 72 hours, we look at v_8 and v_{12}



We see that we have reached an equilibrium value of approximately 1234.57 mg.